Modelling in Mechanics Cheat Sheet

Constructing a model

Mechanics deals with motion and action of forces on objects. Mathematical models can be constructed to simulate real-life situations, but in many cases it is necessary to simplify the problem by making assumptions so that it can be described using equations or graphs in order to solve it.

Example 1: The motion of a basketball as it leaves a player's hand and passes through the net can be modelled using the equation $h = 2 + 1.1x - 0.1x^2$, where h m is the height of the basketball above the ground and x m is the horizontal distance travelled.

a. Find the height of the basketball :

i. When it is released

$x = 0\,; h = 2 + 0 + 0$
$Height = 2\text{m}$

ii. At a horizontal distance of 0.5m

$x = 0.5\,; h = 2 + 1.1 \times 0.5 - 0.1 \times (0.5)^2$
$Height = 2.525\text{ m}$

b. Use the model to predict the height of the basketball when it is at a horizontal distance of 15m from the player.

$x = 15\,; h = 2 + 1.1 \times 15 - 0.1 \times (15)^2$
$Height = -4\text{ m}$

c. Comment on the validity of this prediction.

Height cannot be negative so the model is not valid when $x = 15$ m.

Modelling assumptions

Modelling assumptions can simplify a problem and allow you to analyse the real-life situation using known mathematical techniques. These assumptions will affect the calculations in a particular problem.

Some common models and modelling assumptions

Model	Modelling assumptions
Smooth surface	Assume there is no friction between the surface and any object on it
Rough surface	Objects in contact with the surface experience a frictional force if they are moving or are acted on by a force
Air resistance – Resistance experienced as an object moves through the air	Usually modelled as being negligible
Gravity – Force of attraction between all objects. Acceleration dur to gravity is denoted by g, where the value of g= 9.8 ms^{-2}	• Assume that all objects with mass are attracted towards the Earth • Earth's gravity is uniform and acts vertically downwards • g is constant and is taken as 9.8 ms^{-2}, unless otherwise stated in the question

Quantities and units

The International System of Units, (abbreviated as SI) is the modern form of the metric system.

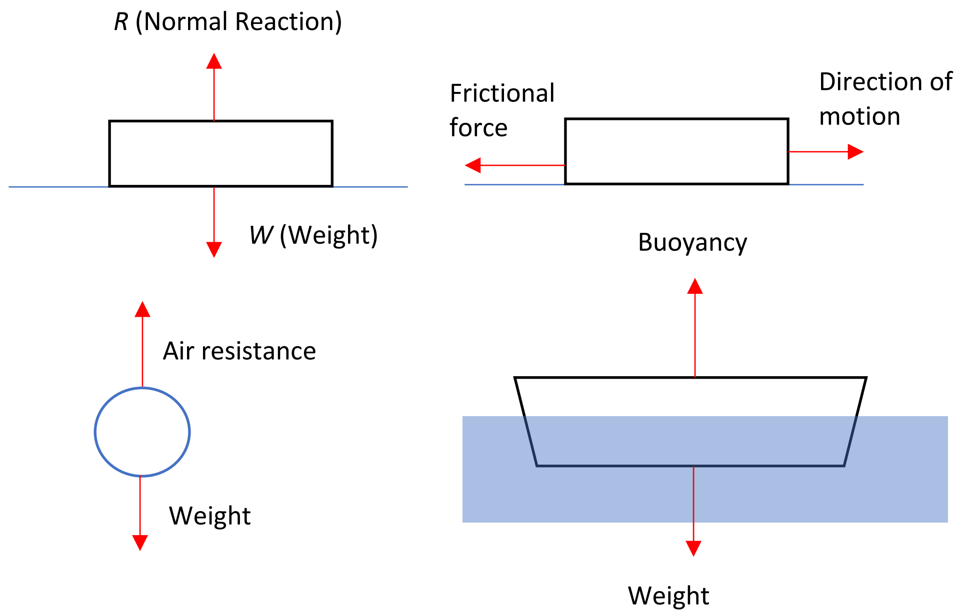
These **base** SI units are most commonly used in mechanics.

Quantity	Unit	Symbol
Mass	Kilogram	kg
Length/displacement	Metre	m
Time	Seconds	s

These **derived** units are compound units built from the base units.

Quantity	Unit	Symbol
Speed/velocity	Metres per second	ms^{-1}
Acceleration	Metres per second per second	ms^{-2}
Weight/force	Newton	N (= kg ms^{-2})

Some of the common force diagrams that you will encounter in mechanics :



Meanings of each of the above forces:

- The **weight** (or gravitational force) of an object acts vertically downwards
- The **normal reaction** is the force acting perpendicular to a surface when an object is in contact with the surface.
- The **friction** is a force which opposes the motion between two rough surfaces
- **Buoyancy** is the upward force on a body that allows it to float or rise when submerged in a liquid.
- **Air resistance** opposes motion of an object falling towards the ground.

Example 2: Write the following quantities in SI units.

a. 4km
$4\text{ km} = 4 \times 1000 = 4000\text{ m}$

b. 0.32 grams
$0.32\text{ g} = 0.32 \div 1000 = 3.2 \times 10^{-4}\text{ kg}$

c. $5.1 \times 10^6\text{ km h}^{-1}$
$5.1 \times 10^6\text{ km h}^{-1} = 5.1 \times 10^6 \times 1000$
$= 5.1 \times 10^9\text{ m h}^{-1}$
$5.1 \times 10^9 \div (60 \times 60) = 1.42 \times 10^6\text{ m s}^{-1}$

Working with vectors

Vector quantities are quantities which have both magnitude and direction. Vector quantities can be positive or negative. Examples are:

Quantity	Description	Unit
Displacement	Distance in a particular direction	Metre (m)
Velocity	Rate of change of displacement	Metres per second (ms^{-1})
Acceleration	Rate of change of velocity	Metres per second per second (ms^{-2})

Scalar quantities are quantities which have magnitude only. Scalar quantities are always positive. Examples are:

Quantity	Description	Unit
Distance	Measure of length	Metre (m)
Speed	Measure of how quickly a body moves	Metres per second (ms^{-1})
Time	Measure of ongoing events taking place	Second (s)
Mass	Measure of the quantity of matter contained in an object	Kilogram (kg)

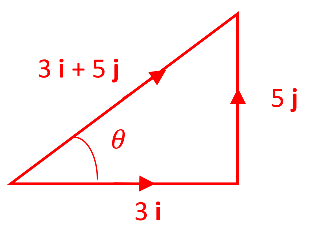
You can also describe vectors using **i-j** notation, where **i** and **j** are the unit vectors in the positive x and y directions.

Example 3: The velocity of a particle is given by v = 3**i** + 5**j** ms^{-1}. Find:

a. The speed of the particle

$|\text{speed}| = |\text{v}| = \sqrt{3^2 + 5^2} = \sqrt{34}$
$= 5.83\text{ ms}^{-1}$

b. The angle the direction of motion of the particle makes with the unit vector **i**



Angle made with i = θ

$\tan\theta = \frac{5}{3}\;\; so\;\theta = 59°$